

Ref #	Hits	Search Query	DBs	Default Operator	Plurals	Time Stamp
L1	2	encrypt\$4.ab. and optical.ab. and decrypt\$5.ab. and algebraic.ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 17:32
L2	3	("6125183").URPN.	USPAT	OR	ON	2005/06/11 17:32
L3	2	encrypt\$4.ab. same optical.ab. same decrypt\$5.ab. same algebraic.ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 17:32
L4	2	encrypt\$4 same optical same decrypt\$5 same algebraic	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 17:33
L5	42	encrypt\$4 same decrypt\$5 same algebraic	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 17:33

Ref #	Hits	Search Query	DBs	Default Operator	Plurals	Time Stamp
L1	1840	encrypt\$5.ab. and decrypt\$5.ab. and (image or video or television or movie or film\$5).ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:45
L2	1101317	(image or video or television or movie or film\$5) with (break\$5 or divid\$5 or partition\$5 or part\$5 or divis\$5 or segment\$5 or block\$5 or piec\$5)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:46
L3	3849	encrypt\$5 with (algebr\$5 or discreet\$5 or cyclotom\$5 or circul\$5 or ellipt\$5 or polynom\$5 or integer or power\$5 or mathmat\$5 or formul\$5)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:47
L4	66	L1 and L2 and L3	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:48
L5	34	L4 and @ad<"20010330"	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:48
L6	1	("6570990").URPN.	USPAT	OR	ON	2005/06/11 15:53
L7	5	("6240183").URPN.	USPAT	OR	ON	2005/06/11 15:56
L8	3	("6064738").URPN.	USPAT	OR	ON	2005/06/11 15:59
L9	0	("5974144").URPN.	USPAT	OR	ON	2005/06/11 16:01
L10	2	("5933499").URPN.	USPAT	OR	ON	2005/06/11 16:02
S1	2	("6505299").PN.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	OFF	2005/06/11 15:44
S2	2	("6505299").URPN.	USPAT	OR	ON	2005/06/10 14:46
S3	70857	encrypt\$5	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:17
S4	1212028	digital	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:17

S6	3210433	image or picture or video or movie	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:18
S7	8424069	polynomial or "one or more" or power or integral or two	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:18
S8	2368095	cyclotom\$4 or circul\$4 or ellipt\$6 or curv\$5	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:19
S9	31738	decrypt\$5	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:19
S10	1	S3 with S4 with S6 with S7 with S8	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/10 15:21
S11	9	S3 same S4 same S6 same S7 same S8	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 12:51
S12	30	cyclotom\$4 with polynom\$5	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:34
S13	3210433	image or picture or video or movie	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:14
S14	7267	encrypt\$5 with S13	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:14
S15	5	S14 and S12	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:15

S16	100	S14 and cinema	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:16
S17	83	S16 and encrypt\$5 and decrypt\$5	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:16
S18	26	S17 and @ad<"20000331"	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:16
S19	5	("6141530").URPN.	USPAT	OR	ON	2005/06/11 13:22
S20	3210433	image or picture or video or movie	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:42
S21	2776	(encrypt\$5 with S20) and (decrypt\$5 with S20)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 13:44
S22	66	S21 and ((mathmat\$6 or algebr\$5 or formul\$5 or unity or discreet) with (encrypt\$5 or algorith\$5))	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 14:39
S23	52	("5862218").URPN.	USPAT	OR	ON	2005/06/11 13:53
S24	2	("5933499").URPN.	USPAT	OR	ON	2005/06/11 13:58
S25	1	("6456985").URPN.	USPAT	OR	ON	2005/06/11 13:58
S26	1	("6363210").URPN.	USPAT	OR	ON	2005/06/11 14:00
S27	1	("6490352").URPN.	USPAT	OR	ON	2005/06/11 14:34
S28	1684	S20.ab. and encrypt\$5.ab. and decrypt\$5.ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 14:35
S29	1692	(S20.ab. or imaging.ab.) and encrypt\$5.ab. and decrypt\$5.ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 14:36

S30	395590	S20 with (partition\$5 or divid\$5 or separat\$5 or segment\$5 or block\$5)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:13
S31	50	S29 and S30 and (mathmat\$6 or algebr\$5 or formul\$5 or unity or discreet)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:15
S32	3210433	image or picture or video or movie	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:12
S33	1919	encrypt\$5.ab. and decrypt\$5.ab. and (S32 or television or T).ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:15
S34	585958	(movie or television or image or video or TV of film\$5) with (partition\$5 or divid\$5 or separat\$5 or segment\$5 or block\$5)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:14
S35	1914	encrypt\$5.ab. and decrypt\$5.ab. and (S32 or television or TV or film\$4).ab.	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:15
S36	383	S34 and S35	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:16
S37	131	S36 and (mathmat\$6 or algebr\$5 or formul\$5 or unity or discreet or power or integral)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:18
S38	147	S36 and (mathmat\$6 or algebr\$5 or formul\$5 or unity or discreet or power or integral or integer)	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:18
S39	85	S38 and @ad<"20010330"	US-PGPUB; USPAT; EPO; JPO; DERWENT; IBM_TDB	OR	ON	2005/06/11 15:18
S40	10	("6195368").URPN.	USPAT	OR	ON	2005/06/11 15:31

S41	1	("6192127").URPN.	USPAT	OR	ON	2005/06/11 15:36
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root of unity



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 root of unity

In mathematics, the ***n*-th roots of unity** or **de Moivre numbers**, named after Abraham de Moivre (1667 - 1754), are complex numbers located on the unit circle. They form the vertices of a *n*-sided regular polygon with one vertex on 1.

Definition

For a given *n* the complex numbers *z* which solve

$$z^n = 1 \quad (n \in \mathbb{N}_0)$$

are called the ***n*-th roots of unity**. There are *n* different *n*-th roots of unity.

The *n*-th roots of unity form a cyclic group of order *n* under multiplication with 1 as the identity element. A generator for this cyclic group is called **primitive *n*-th root of unity**.

Examples

The third roots of unity are

$$\left\{1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}\right\}.$$

The primitive third roots of unity are

$$\left\{\frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}\right\}.$$

The fourth roots of unity are

$$\{1, +i, -1, -i\}.$$

The primitive fourth roots of unity are

$$\{+i, -i\}.$$

Properties

As a consequence of Euler's identity the n -th roots of unity can be written as

$$e^{2\pi i k/n} \quad (k, n \in \mathbb{N}_0 \text{ and } k < n).$$

As long as n is at least 2, these numbers add up to 0, a simple fact that is of constant use in mathematics. It can be proved in any number of ways, for example by recognising the sum as coming from a geometric progression.

The primitive n -th roots of unity are precisely the numbers of the form $\exp(2\pi i k/n)$ where k and n are coprime. Therefore, there are $\varphi(n)$ different primitive n -th roots of unity, where $\varphi(n)$ denotes Euler's phi function. These different roots of unity can be arranged to form the elements of a unitary matrix, and are thus orthogonal to each other. A detailed exposition of the orthogonality relationship is given in the article character group.

Cyclotomic polynomials

The n -th roots of unity are precisely the zeros of the polynomial $p(X) = X^n - 1$; the primitive n th roots of unity are precisely the zeros of the n th cyclotomic polynomial

$$\Phi_n(X) = \prod_{k=1}^{\varphi(n)} (X - z_k)$$

where $z_1, \dots, z_{\varphi(n)}$ are the primitive n -th roots of unity. The polynomial $\Phi_n(X)$ has integer coefficients and is irreducible over the rationals (i.e., cannot be written as a product of two positive-degree polynomials with rational coefficients). The case of prime n , which is easier than the general assertion, follows from Eisenstein's criterion.

Every n th root of unity is a primitive d th root of unity for exactly one positive divisor d of n . This implies that

$$X^n - 1 = \prod_{d|n} \Phi_d(X).$$

This formula represents the factorization of the polynomial $X^n - 1$ into irreducible factors and can also be used to compute the cyclotomic polynomials recursively. The first few are

$$\begin{aligned}\Phi_1(X) &= X - 1 \\ \Phi_2(X) &= X + 1 \\ \Phi_3(X) &= X^2 + X + 1 \\ \Phi_4(X) &= X^2 + 1 \\ \Phi_5(X) &= X^4 + X^3 + X^2 + X + 1 \\ \Phi_6(X) &= X^2 - X + 1\end{aligned}$$

In general, if p is a prime number, then all p th roots of unity except 1 are primitive p th roots, and we have

$$\Phi_p(X) = \frac{X^p - 1}{X - 1} = \sum_{k=0}^{p-1} X^k$$

Note that, contrary to first appearances, *not* all coefficients of all cyclotomic polynomials are 1, -1, or 0; the first polynomial where this occurs is Φ_{105} , since $105 = 3 \times 5 \times 7$ is the first product of three odd primes.

Cyclotomic fields

By adjoining a primitive n th root of unity to \mathbf{Q} , one obtains the **n th cyclotomic field** F_n . This field contains all n th roots of unity and is the splitting field of the n th cyclotomic polynomial over \mathbf{Q} . The field extension F_n/\mathbf{Q} has degree $\varphi(n)$ and its Galois group is naturally isomorphic to the multiplicative group of units of the ring $\mathbf{Z}/n\mathbf{Z}$.

As the Galois group of F_n/\mathbf{Q} is abelian, this is an abelian extension. Every subfield of a cyclotomic field is an abelian extension of the rationals. In these cases Galois theory can be written out quite explicitly

in terms of Gaussian periods: this theory from the *Disquisitiones Arithmeticae* of Gauss was published many years before Galois.

Conversely, *every* abelian extension of the rationals is such a subfield of a cyclotomic field - a theorem of Kronecker, usually called the Kronecker-Weber theorem on the grounds that Weber supplied the proof.

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Best of the Web

Some good "**root of unity**" pages on the web:



Math

mathworld.wolfram.com

Mentioned In

root unity is mentioned in the following topics:

Chowla-Mordell theorem
trigonometric interpolation
abelian extension
Kronecker-Weber theorem
unit (ring theory)

primitive root
Weil pairing
conjugate element (field theory)
List of algebraic number theory topics
Iwasawa theory
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